SALT AND NEARs Locality

Progression in Calculations Policy

January 2017

To be reviewed annually   next review Sep 2018

Before children move to written methods, they need:

- To understand the number system
- Know some number facts
- Have good mental strategies / mental agility!
- Be confident use concrete apparatus and pictorial representations to solve problems and explain their reasoning.

When children move to written methods they need to think...

- What will the answer be roughly?
- Can I work it out in my head?
- What can I use to help me? Do I need a written method?
- Does that answer my question?
- Does it make sense? Can I check?

Purpose of the Policy:

- To make teachers and parents aware of the strategies that pupils are formally taught within each year group that will support them to perform mental and written calculations. Pupils should not move on through the methods until they have secured and understood how to use the methods, including the concrete and pictorial representations.
- The policy supports teachers in identifying appropriate concrete apparatus and pictorial representations to help develop and secure understanding.

Aims of the policy:

- To ensure consistency and progression in our approach to calculation.
- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations.
- To ensure that children can use these methods accurately with confidence and understanding.

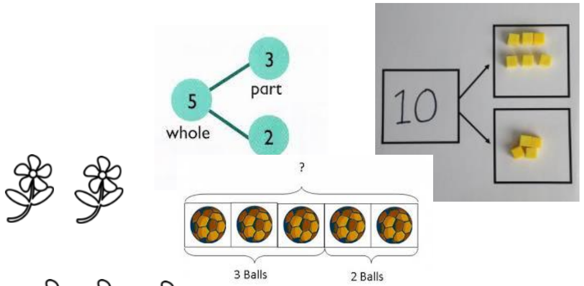
How to use this policy:

- Use the policy as the basis of your planning but ensure you use previous or following years' guidance to allow for personalised learning.
- Always use Assessment for Learning to identify suitable next steps in calculation for groups of children.
- If, at any time, children are making significant errors, return to the previous stage in calculation.
- Always introduce a new concept/calculation using use suitable resources, models and images to support children's understanding of the calculation and place value, as appropriate.
- Encourage children to make sensible choices about the methods they use when solving problems.

Add Plus Total + Addition + More Sum Altogether

Year R
Method to be used by core of class

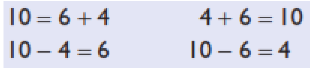
Use pictures, tens frames, cubes and other concrete resources to to add two numbers together as a group or in a bar.

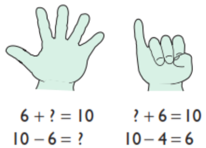


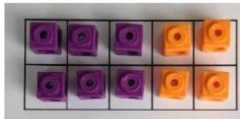
See addition appendix 1- combining two parts to make a whole: part-whole model.

Year 1
Method to be used by core of class

As year R plus:
Teach all the number bonds up to and including 10 and the related 'Fact Family' for each fact.



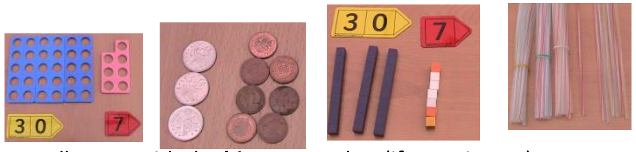




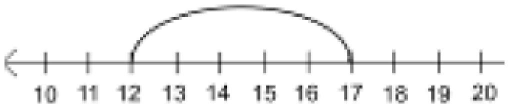
Use concrete objects to combine groups to add and solve missing number problems.

3+__= 10 Show this using the part/whole model.

Understand place value – can partition numbers and recombine numbers



Usually start with the **biggest** number (if counting on)
12 + 5 = 17

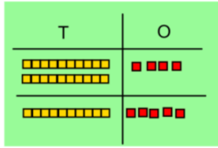


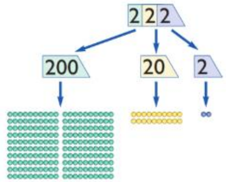
Start at the larger number on the number line and count on in ones or in one jump to find the answer.

See addition appendix 1- combining two parts to make a whole: part-whole model. Appendix 2 starting at the bigger number and counting on.

Year 2
Method to be used by core of class

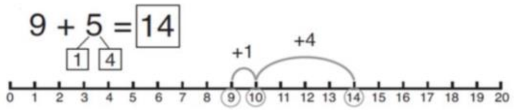
As year 1 plus:
Addition can be done in any order (commutative)
34 + 56 or 56 + 34
Understand place value – can partition numbers & recombine numbers



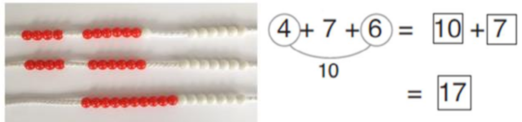


37= 30 + 7 30+7 = 37

Use partitioning to add numbers, first with concrete apparatus, then as a possible mental method.
Have a range of mental methods for calculating first with numbers to 20, then with numbers to 100 e.g. breaking numbers apart to use them flexibly, this may be with a bridging strategy (e.g. 7+5 could be thought of as 7+3+2 or 5+5+2), a compensating strategy (e.g. 7+9 could be thought of as 7+10 then -1) or by using a near double (e.g. 7+8 =14+1).



Learn to add three numbers 4 + 7 + 6= 17
Put 4 and 6 together to make 10. Add on 7.



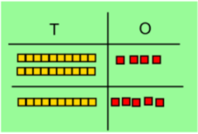
Use number bonds e.g. 4+6=10 to work out 40+60=100

See addition appendix 2 starting at the bigger number and counting on. Appendix 3 regrouping to make 10. Appendix 4 adding three single digits.

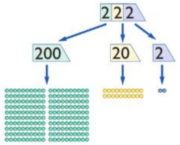
Add Plus Total + Addition + More Sum Altogether

Year 3
Method to be used by core of class

As year 2 plus:



Understand place value – can partition numbers & recombine numbers to support column addition.



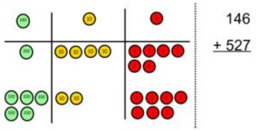
24 + 15=

Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.

Expanded addition, TU then TU crossing tens barriers, then HTU (three digits)

34 + 62=

30 + 4
60 + 2
90 + 6 = 96



494 + 368 =

400 + 90 + 4
300 + 60 + 8
700 +150 +12 =862

then Compact addition

494
+368
862
1 1

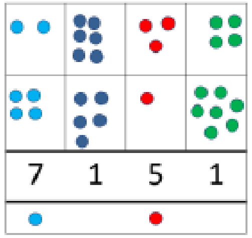
See addition appendix 5 column method- no regrouping and appendix 6 column method – regrouping (bridging ten)

Year 4
Method to be used by core of class

As year 3 plus:

Add ones, tens and hundreds to a three-digit number

Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.



Compact addition (integers only) with numbers up to four digits

e.g.

7648
+ 1486
9134
1 1 1

Expanded addition may be used for decimals in real contexts e.g. money and length.

£11.35+ £12.43=

£10 + £1 + 30p + 5p +
£10 + £2 + 40p + 3p
£20 + £3 + 70p + 8p = £23.78

See addition appendix 5 column method- no regrouping and appendix 6 column method – regrouping (bridging ten)

Year 5
Method to be used by core of class

As year 4 plus:

Compact addition with numbers larger than four digits.

Compact addition with decimals to two places.

e.g.

32.75
+48.64
81.39
11

	2	3	.	3	6	1
		9	.	0	8	0
	5	9	.	7	7	0
+		1	.	3	0	0
	9	3	.	5	1	1
	2	1		2		

See addition appendix 5 column method- no regrouping and appendix 6 column method – regrouping (bridging ten)

Year 6
Method to be used by core of class

As year 5 plus:

Compact addition involving large numbers.

Compact addition with decimals to three places.

e.g.

32.756
+48.646
81.402
1 1 1 1

24.5+ 36.238

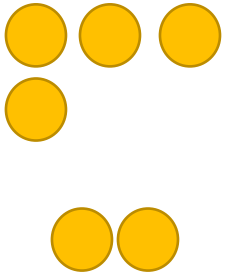
24.500
+36.238
60.738
1

See addition appendix 5 column method- no regrouping and appendix 6 column method – regrouping (bridging ten)

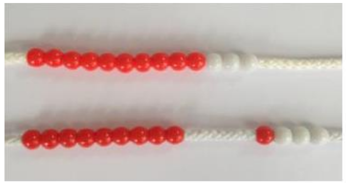
Subtract take away less than **- Subtraction -** minus difference between

Year R
Method to be used by core of class

Use physical objects, counters, cubes etc to show how objects can be taken away.



6 – 2 = 4

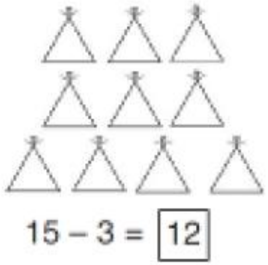


Use counters and bead strings, move them away from the group as you take them away counting backwards as you go.

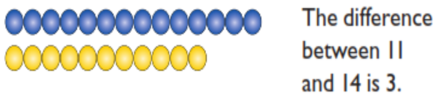
See subtraction appendix 1 taking away ones and appendix 2 counting back.

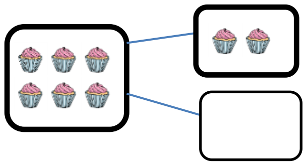
Year 1
Method to be used by core of class

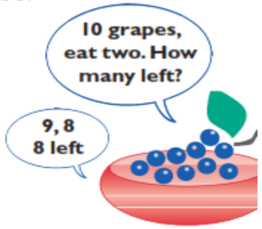
As year R plus:



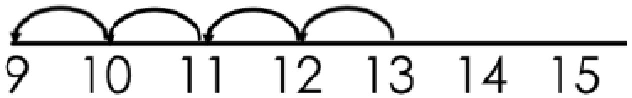
Understand that subtraction can be seen as taking away and finding the difference. Use the part-whole model to take away.







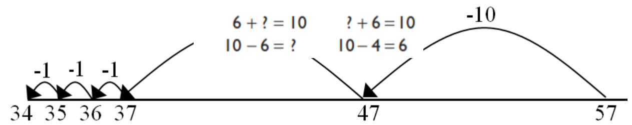
First with concrete apparatus, then number l
mentally. Count back on a number line or number track when secure with concrete apparatus.



See subtraction appendix 1 taking away ones, appendix 2 counting back, appendix 3 finding the difference and appendix 4 part-whole model.

Year 2
Method to be used by core of class

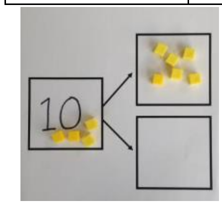
As year 1 plus:
Subtract using concrete objects such as Numcion, make the whole and take away the correct amount. Then progress to pictorial representations and mental methods.
Start at the bigger n the smaller number
showing the jumps

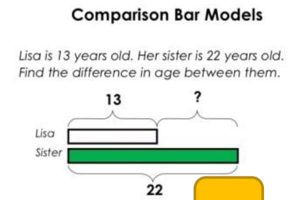


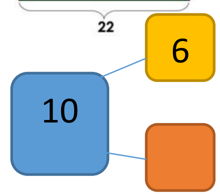
This can progress all the way to counting back using two 2 digit numbers.

No. bonds to 100 (at least with multiples of 10). Understand the number line as a continuum. Understand that subtraction is the inverse of addition (Numicon is a particularly useful image) and bar model.

10	
6	4





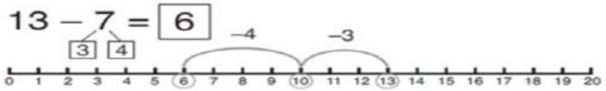


See subtraction appendix 2 counting back, appendix 3 finding the difference and appendix 4 part-whole model and appendix 5 make 10.

Subtract   take away   less than   **- Subtraction -**   minus   difference between

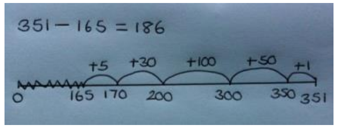
Year 3
Method to be used by core of class

As year 2 plus:

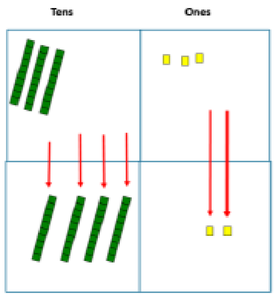


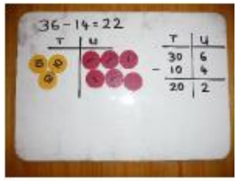
Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.

Number line method (2 and 3 digit numbers)
351-165=186



Begin expanded subtraction using concrete objects and pictorial representations.

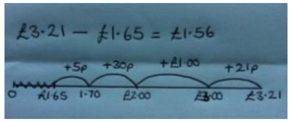




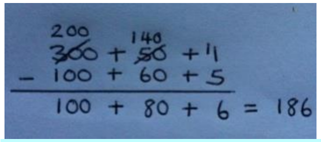
See subtraction appendix 5 make 10. Appendix 6 column method without regrouping.

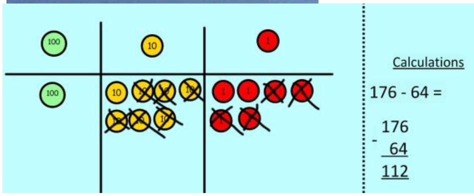
Year 4
Method to be used by core of class

As year 3 plus:
Number line method (2, 3, 4 digit numbers, extending to decimals in a real context)
e.g.



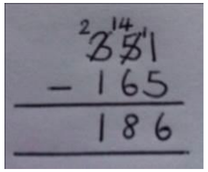
Expanded subtraction
e.g. 354 – 165





Use base 10 or place value counters alongside the written calculation to help to show working.

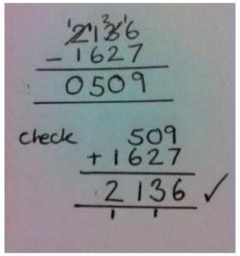
Compact subtraction



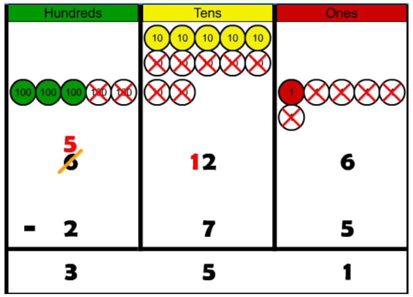
See subtraction appendix 5 make 10. Appendix 6 column method without regrouping.

Year 5
Method to be used by core of class

As year 4 plus:
Compact subtraction, involving numbers larger than 4 digits and with decimals to 2 places.



Draw the counters onto a place value grid and

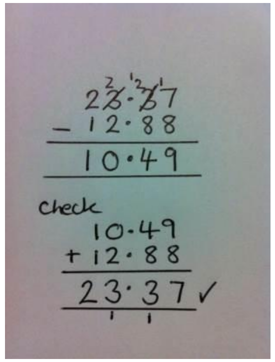


show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. When confident children can find their own way to record the exchange/regrouping.

See subtraction appendix 5 make 10. Appendix 6 column method without regrouping.

Year 6
Method to be used by core of class

As year 5 plus:
Compact subtraction involving large numbers.
Compact subtraction with decimals up to three places.

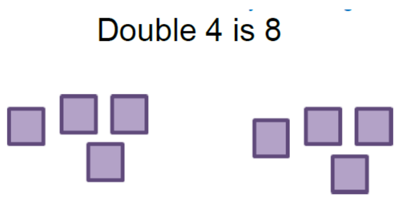


See subtraction appendix 5 make 10. Appendix 6 column method without regrouping.

Multiply times lots of **x Multiplication x** groups of multiple of product

Year R
Method to be used by core of class

Introduce language and concept of making equal groups.
Begin to double numbers to 5. Use concrete apparatus to show how to double a number.

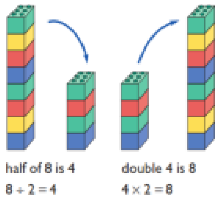


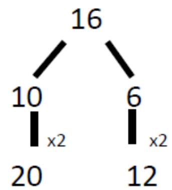
See multiplication appendix 1 doubling.

Year 1
Method to be used by core of class

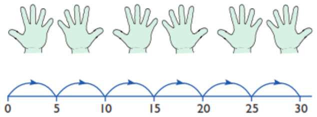
As year R plus:

Recall doubles to 10. Use this knowledge to support halving and doubling larger numbers.



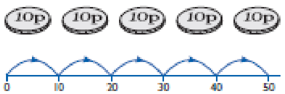


Understand multiplication as repeated addition.



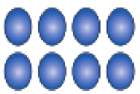
$5+5+5+5+5+5=30$
$5 \times 6 = 30$
5 multiplied by 6
6 groups of 5
6 hops of 5

$2+2+2+2+2=10$



Group sets of objects reliably in 2s, 5s and 10s.
Recognise number sequences e.g. 2s, 5s and 10s.

Use of arrays



$4 \times 2 = 8$

$2 \times 4 = 8$

See multiplication appendix 1 doubling. Appendix 2 counting in multiples. Appendix 3 repeated addition. Appendix 4 arrays- showing commutative multiplication.

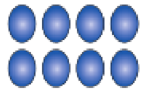
Year 2
Method to be used by core of class

As Year 1 plus:
By the end of the year pupils should recall all multiplication facts for the 2, 5 and 10 times tables.

Understand multiplication as scaling.

The giant is twice as big as a boy.

Understand that multiplication is commutative (arrays eg. Numicon and Cuisenaire particularly useful).



$4 \times 2 = 8$

$2 \times 4 = 8$

Understand that multiplication and division are the inverse of each other.

4x10=40
10x4=40
40÷4=10
40÷10=4

See multiplication appendix 2 counting in multiples. Appendix 3 repeated addition. Appendix 4 arrays- showing commutative multiplication.

Multiply times lots of **x Multiplication x** groups of multiple of product

Year 3
Method to be used by core of class

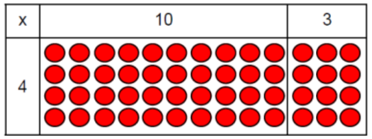
As year 2 plus:
Focus on understanding, representing and remembering times tables facts for 2,5,10,3,4 and 8 times tables, including division facts

e.g 

4x8=32. 8x4=32, 32÷4=8, 32÷8=4

Note - before moving to any TU x U, the children will need be able to multiply a multiple of 10 by a single digit (T0xU)
Numicon or Cuisenaire in the grid
e.g. 20x4, 40x5

Grid method TU x U or HTU X U
Show the link with arrays to first introduce the grid method.
e.g. 4 x 13



e.g. 7 x 39

X	30	9	Total
7	210	63	273

(but know when to calculate mentally e.g. x2, x10, x5)

See multiplication appendix 4 arrays- showing commutative multiplication. Appendix 5 grid method.

Year 4
Method to be used by core of class

As year 3 plus:
ALL times tables facts to 12 x 12 should be known by end of year 4 including multiplying by 0 and 1. Children should learn to multiply three numbers together.
4 x 6 x 3=
4 x 6= 24 x 3= 72

Grid method TU x U or HTU x U
e.g. 7 x 39

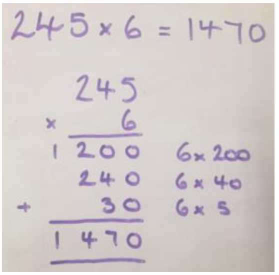
X	30	9	Total
7	210	63	273

(but know when to calculate mentally e.g. x2, x10, x5)

e.g. 245 x 6

x	200	40	5	Total
6	1200	240	30	1470

Moving onto (when ready), Long Multiplication (expanded)



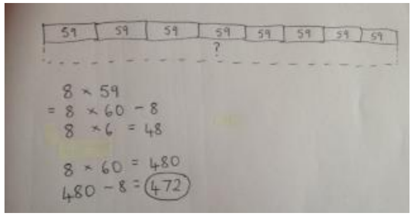
See multiplication appendix 4 arrays- showing commutative multiplication. Appendix 5 grid method.

Year 5
Method to be used by core of class

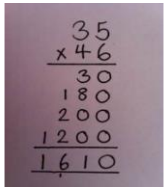
As year 4 plus:
Multiply with numbers up to 4 digits.
Grid Method for TU x TU, HTU x TU, THTU x TU or U.
e.g 35 x 46

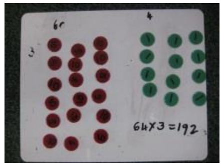
x	30	5	Total
40	1200	200	1400
6	180	30	210
Total			1610

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written method.



Moving on to… Long Multiplication (expanded)

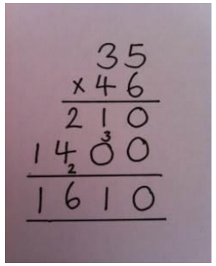




See multiplication appendix 4 arrays- showing commutative multiplication. Appendix 5 grid method. Appendix 6 column multiplication.

Year 6
Method to be used by core of class

As year 5 plus:
Long Multiplication
Up to 4 digit x 2 digit



Moving to… Decimal numbers to 2 places multiplied by whole numbers

Note -some children may continue to use the grid method

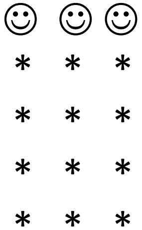
If it helps, children can write out what they are solving next to their answer.

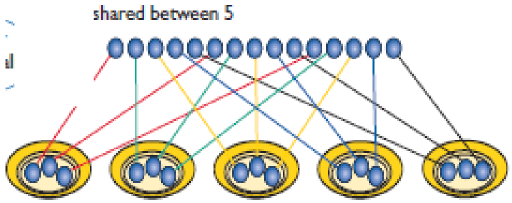
```
   32
 x 24
    8   (4 x 2)
  120   (4 x 30)
   40   (20 x 2)
  600   (20 x 30)
  768
```

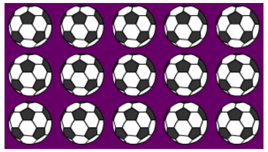
See multiplication appendix 4 arrays- showing commutative multiplication. Appendix 5 grid method. Appendix 6 column multiplication.

Share equally group equally divide **÷ Division ÷** remainder factor quotient

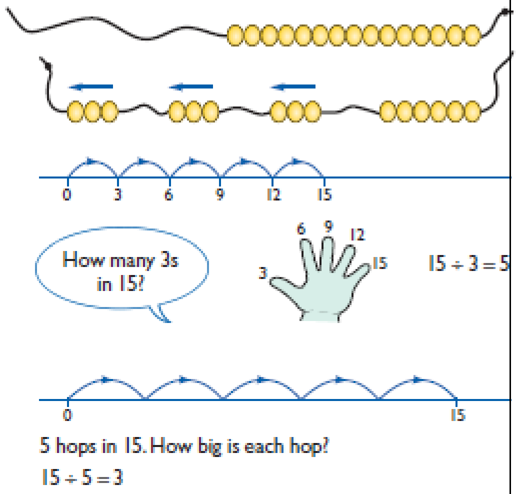
<table>
<tr>
<th>Year R
Method to be used by core of class</th>
<th>Year 1
Method to be used by core of class</th>
<th>Year 2
Method to be used by core of class</th>
</tr>
<tr>
<td>Introduce language and concept of sharing fairly and making equal groups.</td>
<td>Understand division as sharing equally into groups.
Share into groups using concrete apparatus then move to pictorial representations.

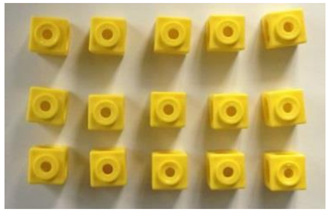





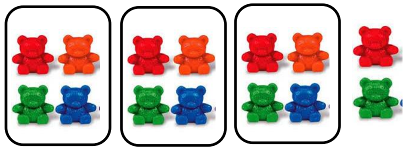
Know multiplication facts (including the related 'fact family' e.g 3x5=15, 5x3=15, 15÷3=5, 15÷5=3)


3x5=15 15÷5=3
5x3=15 15÷3=5
Finding half and quarter using the same methods.

See division appendix 1 sharing objects into groups. Appendix 2 division as grouping. Appendix 3 division within arrays.</td>
<td>As Year 1 plus:
By the end of the year pupils should recall all division facts for the 2, 5 and 10 times tables.



Link division to multiplication by creating an array and thinking about the number sentences that can be created.

Eg 15 ÷ 3 = 5 5 x 3 = 15
15 ÷ 5 = 3 3 x 5 = 15

Finding remainers: Divide objects between groups and see how much is left over 14 ÷ 3 =



See division appendix 1 sharing objects into groups. Appendix 2 division as grouping. Appendix 3 division within arrays. Appendix 4 division with a remainder.</td>
</tr>
</table>

Share equally group equally divide ÷ **Division** ÷ **remainder** **factor** **quotient**

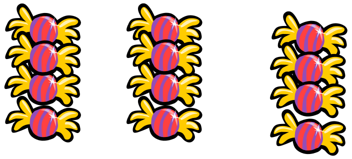
Year 3
Agreed method to be used by core of class

As year 2 plus:
Focus on understanding, representing and remembering times tables facts for 2,5,10,3,4 and 8 times tables, including division facts.

e.g

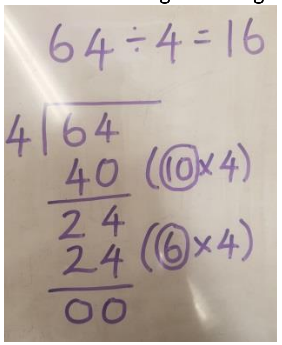
4x8=32. 8x4=32, 32÷4=8, 32÷8=4

It is especially important that children understand that division can be grouping or sharing.
e.g. 12÷3=4
12 sweets between 3 people gives 4 sweets each.



(3 groups of 4)
'How many 3s in 12?' gives 4 groups of 3

Then introducing chunking TU ÷ U

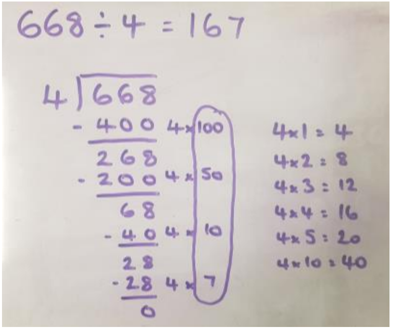


See division appendix 3 division within arrays. Appendix 4 division with a remainder.

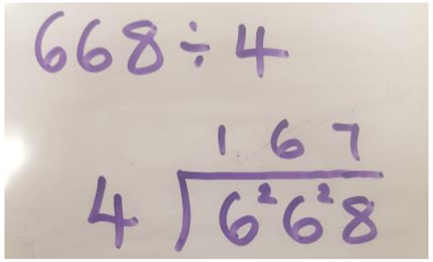
Year 4
Agreed method to be used by core of class

As year 3 plus:
Focus on understanding, representing and remembering times tables facts for ALL times tables up to 12 x12 including division facts.

Continue with chunking for TU ÷ U moving onto HTU ÷ U



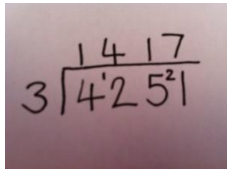
Moving onto short division (when ready) for TU ÷ U and HTU ÷ U



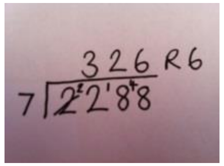
See division appendix 3 division within arrays. Appendix 4 division with a remainder.

Year 5
Agreed method to be used by core of class

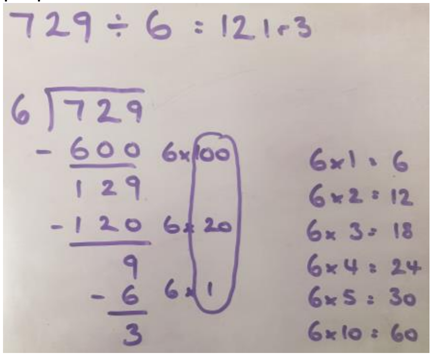
As year 4 plus:
Continue with short division, up to 4 digit numbers divided by 1 digit numbers
e.g 4251÷3



Including dealing with remainders in context.



Or…Chunking to consolidate understanding in preparation for HTU ÷ TU in Year 6



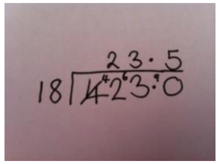
Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.

See division appendix 3 division within arrays. Appendix 4 division with a remainder. Appendix 5 short division.

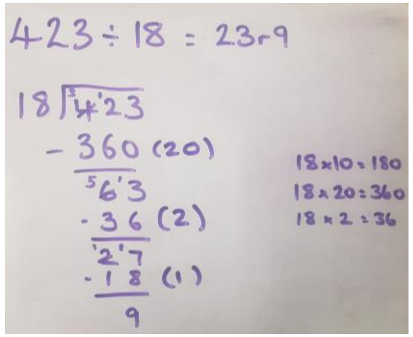
Year 6
Agreed method to be used by core of class

As year 5 plus:
Short division, up to 4 digit numbers divided by 1 or 2 digit numbers including interpreting remainders as decimals and fractions

e.g. 423 ÷ 18



Or Chunking (when dividing by 2 digit numbers)

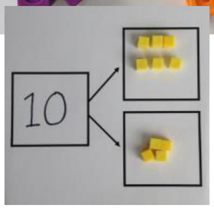
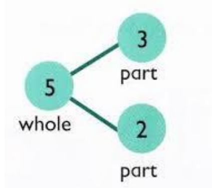
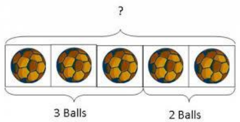
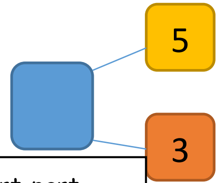
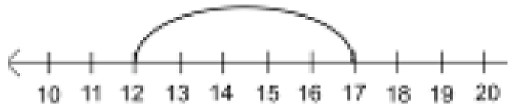


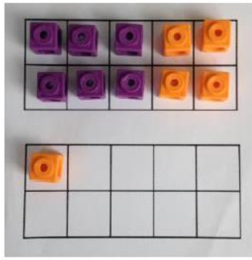
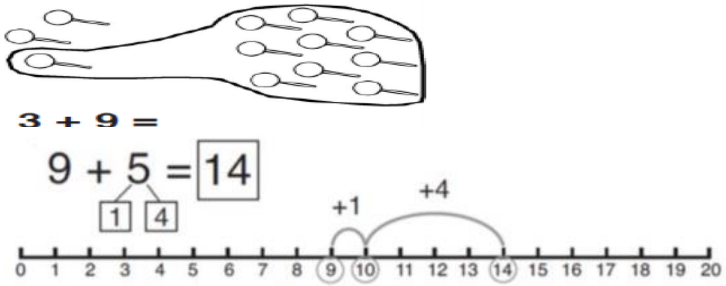
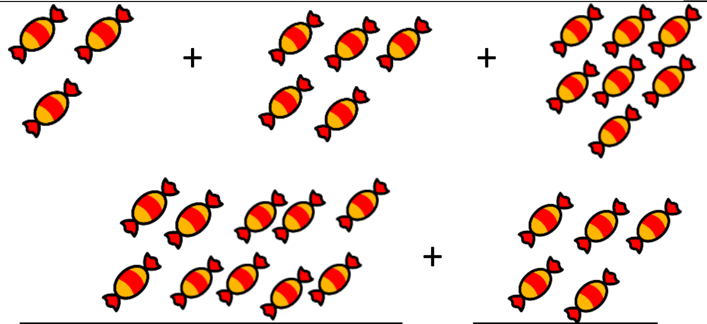
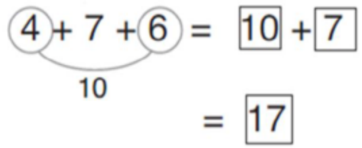
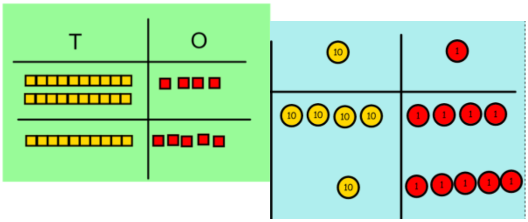
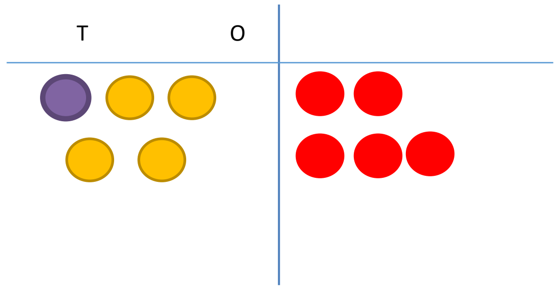
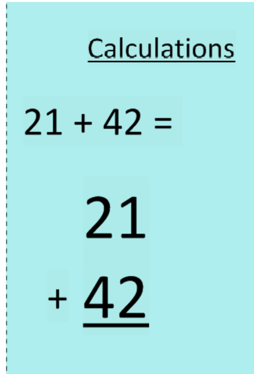
See division appendix 3 division within arrays. Appendix 4 division with a remainder. Appendix 5 short division.

Appendix

Progression in calculations linked to concrete apparatus, pictorial representations and abstract methods. When introducing a new method of calculation the concrete apparatus should be used first. Once this is secure pupils can then be moved onto pictorial representations and then abstract methods.

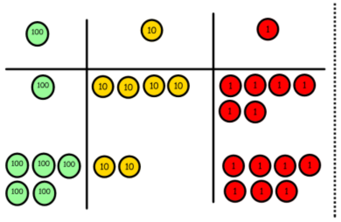
Addition:

Objective and Strategies	Concrete	Pictorial	Abstract
Appendix 1- Combining two parts to make a whole: part- whole model	  Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar.  	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Appendix 2- Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Appendix 3- Regrouping to make 10.	 6 + 5 = 11	Use pictures or a number line. Regroup or partition the smaller number to make 10.	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?

	 Start with the bigger number and use the smaller number to make 10.	 	
Appendix 4- Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.
Appendix 5- Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	After [illegible] dren can draw the counters to help them to solve additions.  	

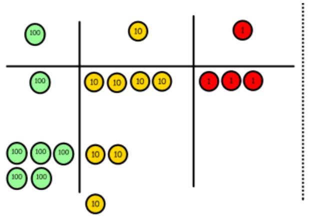
Appendix 6-
Column method- regrouping (bridging 10)

Make both numbers on a place value grid.



$$\begin{array}{r} 146 \\ \underline{+\ 527} \end{array}$$

Add up the units and exchange 10 ones for one 10.



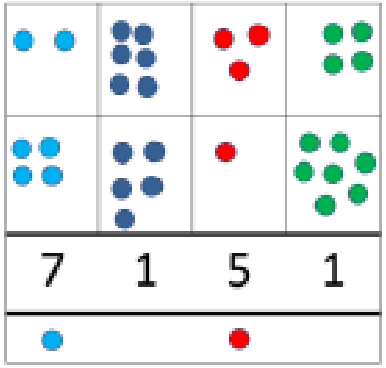
$$\begin{array}{r} 146 \\ \underline{+\ 527} \end{array}$$

Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.



Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

$$\begin{array}{lll} 20 & + & 5 \\ \underline{40} & \underline{+} & \underline{8} \\ 60 & + & 13 \quad = 73 \end{array}$$

$$\begin{array}{r} 536 \\ \underline{+\ 85} \\ \underline{621} \\ 1\,1 \end{array}$$

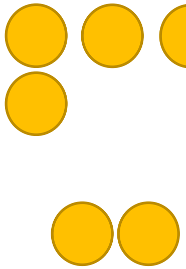
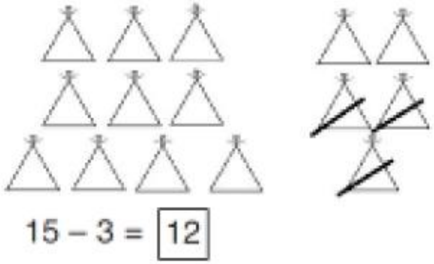
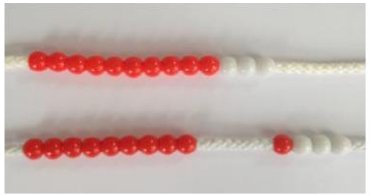
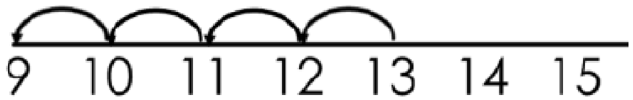
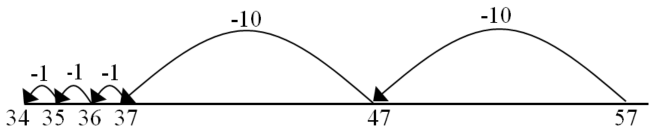
As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

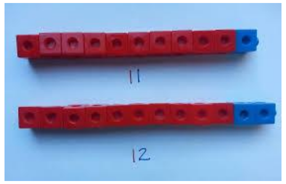
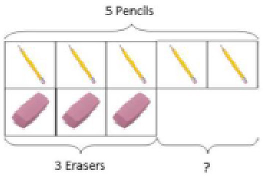
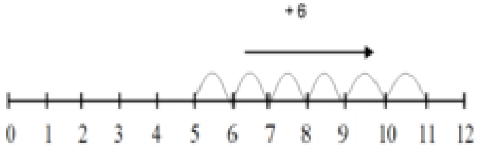
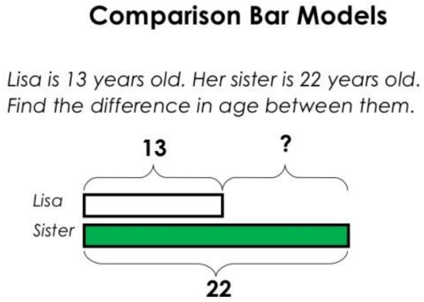
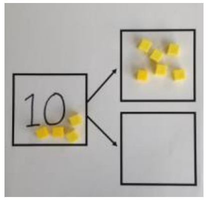
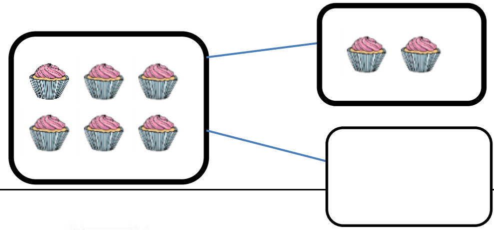
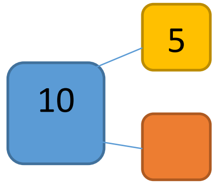
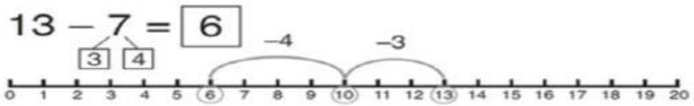
$$\begin{array}{r} 72.8 \\ \underline{+\ 54.6} \\ \underline{127.4} \\ 1\ \ 1 \end{array}$$

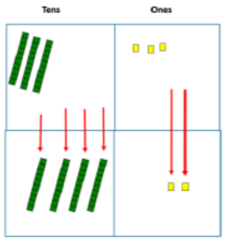
$$\begin{array}{rrrrrrr} & £ & 2 & 3 & . & 5 & 9 \\ + & £ & & 7 & . & 5 & 5 \\ \hline & £ & 3 & 1 & . & 1 & 4 \\ \hline & & 1 & 1 & & 1 & \end{array}$$

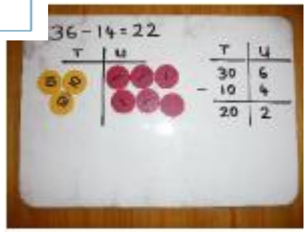
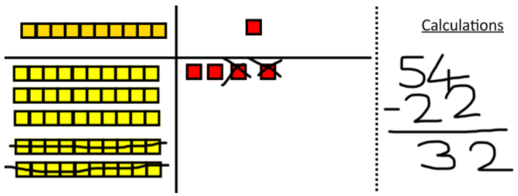
$$\begin{array}{rrrrrr} 2 & 3 & . & 3 & 6 & 1 \\ & 9 & . & 0 & 8 & 0 \\ 5 & 9 & . & 7 & 7 & 0 \\ + & 1 & . & 3 & 0 & 0 \\ \hline 9 & 3 & . & 5 & 1 & 1 \\ \hline 2 & 1 & & 2 & & \end{array}$$

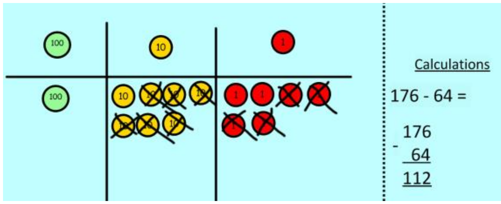
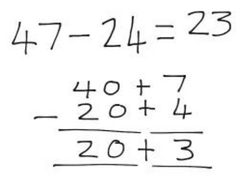
Subtraction:

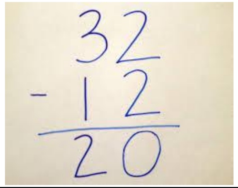
Objective and Strategies	Concrete	Pictorial	Abstract
Appendix 1- Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  - 2 = 4 	Cross out drawn objects to show what has been taken away.  	18 -3= 15 8 – 2 = 6
Appendix 2- Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

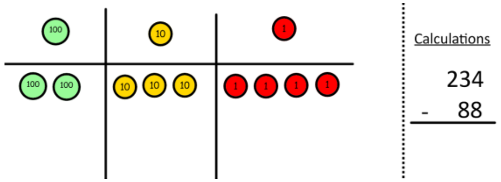
Appendix 3- Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference Use basic bar models with items to find the difference  	 Count on to find the difference.  Draw bars to find the difference between 2 numbers.	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Appendix 4 Part- Whole Model	Link to addition- use the part whole model to help explain the inverse between addition and subtraction.  If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.
Appendix 5- Make 10	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?

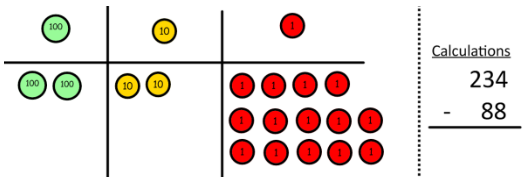
<table>
<tr>
<td>Appendix 6-
Column method without regrouping</td>
<td>Use Base 10 to make the bigger number then take the smaller number away.


Show how you partition numbers to subtract. Again make the larger number first.

</td>
<td>

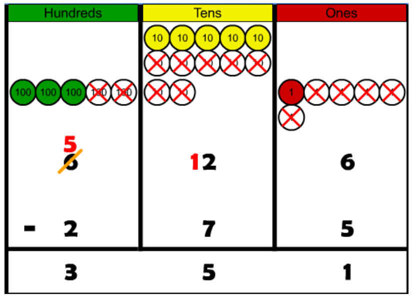
Draw the Base 10 or place value counters alongside the written calculation to help to show working.

</td>
<td>

This will lead to a clear written column subtraction.

</td>
</tr>
<tr>
<td>Appendix 7-
Column method with regrouping</td>
<td>Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

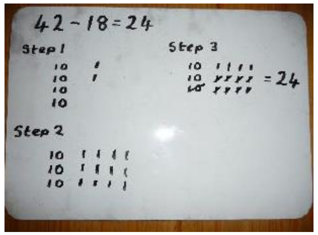
Make the larger number with the place value counters


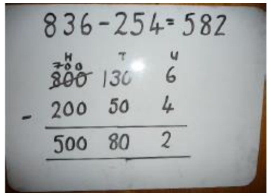
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.


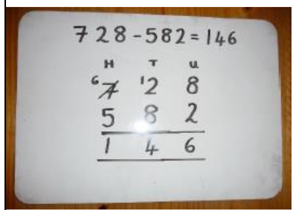
Now I can subtract my ones.

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.</td>
<td>

Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

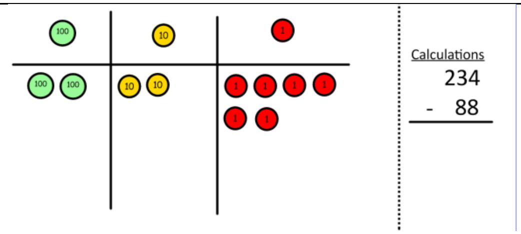
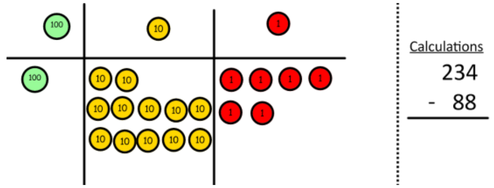
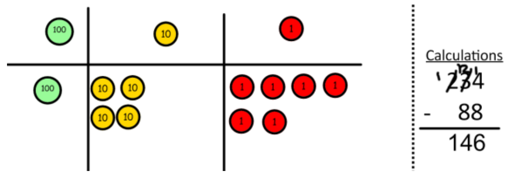
When confident, children can find their own way to record the exchange/regrouping.


Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.</td>
<td>

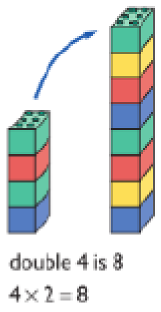
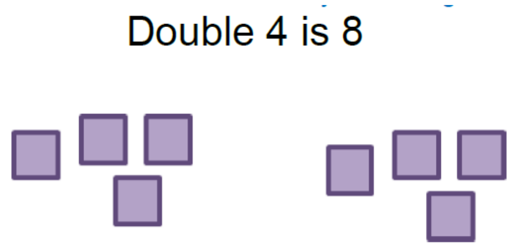
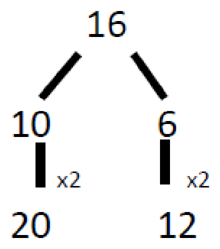
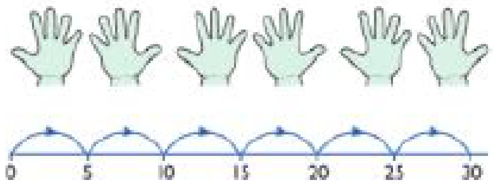
Children can start their formal written method by partitioning the number into clear place value columns.


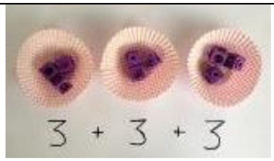
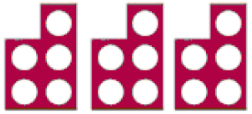
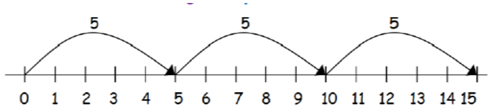
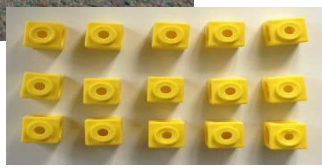
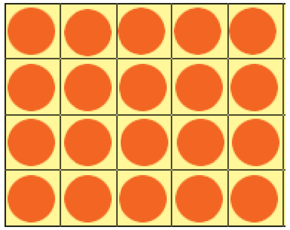
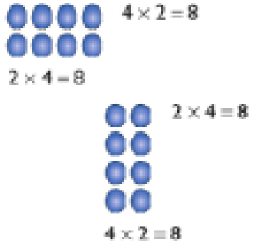
Moving forward the children use a more compact method.

This will lead to an understanding of subtracting any number including decimals.</td>
</tr>
</table>

	  Now I can take away eight tens and complete my subtraction  Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		$\begin{array}{rcccl} & 2 & \overset{5}{\cancel{6}} & \overset{12}{\cancel{3}} & . \overset{1}{0} \\ - & & 2 & 6 & . 5 \\ \hline & 2 & 3 & 6 & . 5 \end{array}$

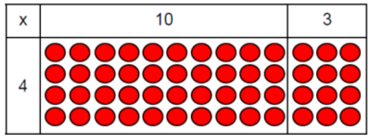
Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Appendix 1- Doubling	Use practical activities to show how to double a number.  	Draw pictures to show how to double a number.  	 Partition a number and then double each part before recombining it back together.
Appendix 2- Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Appendix 3- Repeated addition	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures. 
Appendix 4- Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.  Link arrays to area of rectangles. 	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

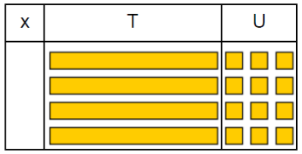
Appendix 5-
Grid Method

Show the link with arrays to first introduce the grid method.



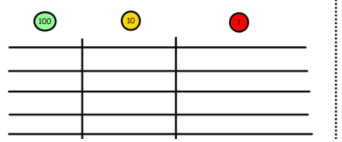
4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.



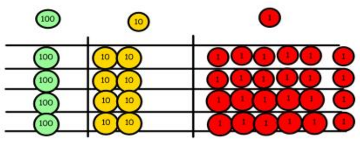
4 rows of 13

Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



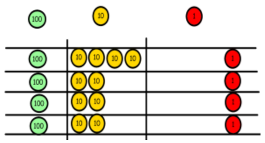
Calculations
4 x 126

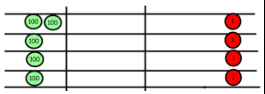
Fill each row with 126.



Calculations
4 x 126

Add up each column, starting with the ones making any exchanges needed.

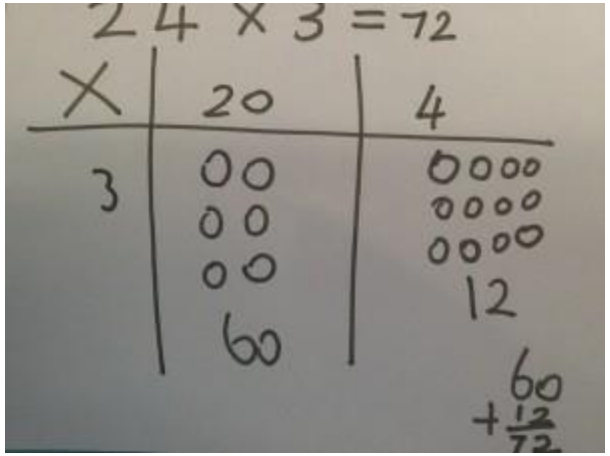




Then you have your answer.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

×	30	5
7	210	35

210 + 35 = 245

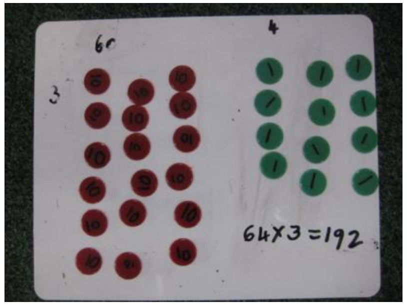
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

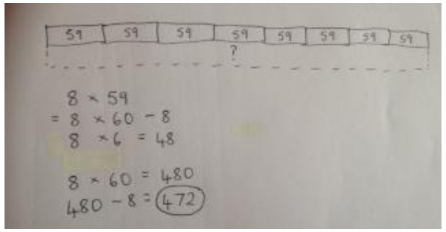
Appendix 6-
Column multiplication

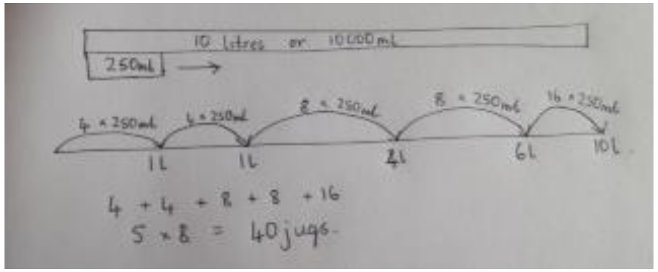
Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.





Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

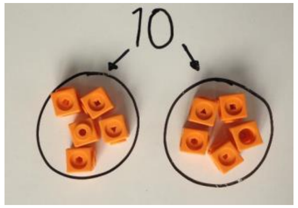
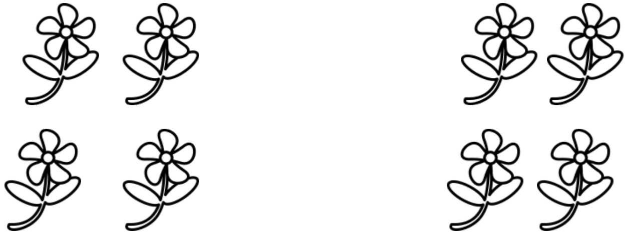
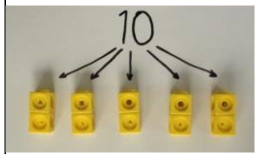
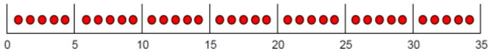
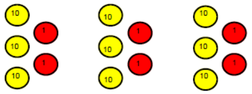
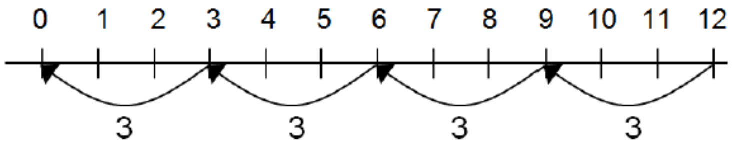
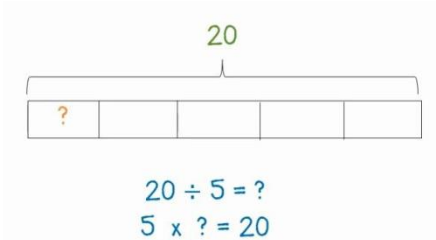
```
   32
x  24
    8   (4 x 2)
  120   (4 x 30)
   40   (20 x 2)
  600   (20 x 30)
  768
```

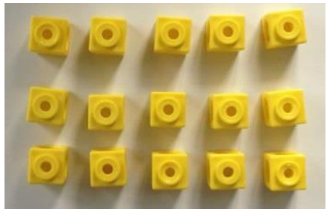
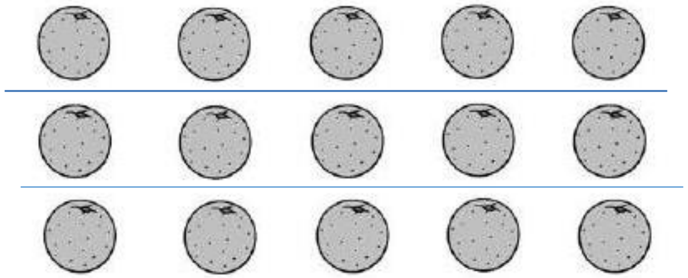
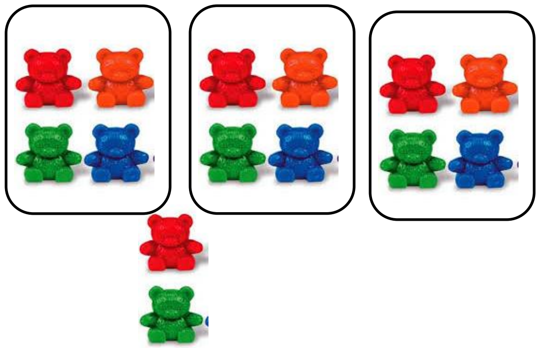
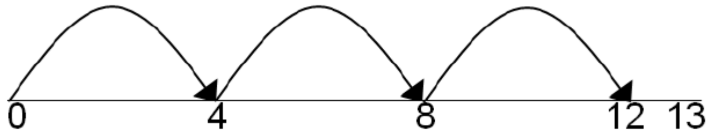
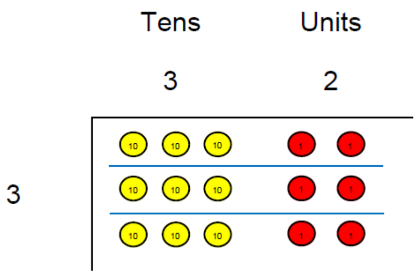
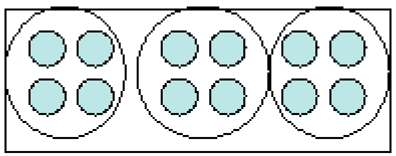
```
      7 4
  x   6 3
      1 2
    2 1 0
    2 4 0
+ 4 2 0 0
  4 6 6 2
```

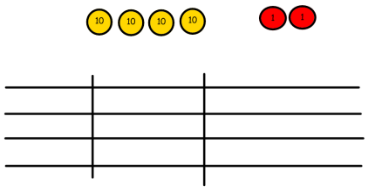
This moves to the more compact method.

```
  2 3   1
   1 3 4 2
x      1 8
 1 3 4 2 0
 1 0 7 3 6
 2 4 1 5 6
   1
```

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Appendix 1- Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$	Share 9 buns between three people. $9 \div 3 = 3$
Appendix 2- Division as grouping	Divide quantities into equal groups.  Use cubes, counters, objects or place value counters to aid understanding.  $96 \div 3 = 32$  	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?

Appendix 3- Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Appendix 4- Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. 	Complete written divisions and show the remainder using r. 29 ÷ 8 = 3 REMAINDER 5 dividend divisor quotient remainder
Appendix 5- Short division	 Use place value counters to divide using the bus stop method alongside	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. Move onto divisions with a 2 1 8 8 6 r 2 3 5 | 4 3 2 remainder.

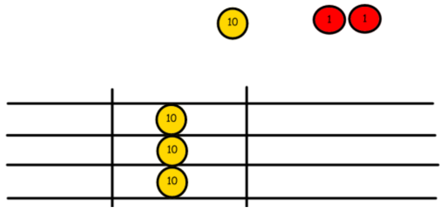


Calculations

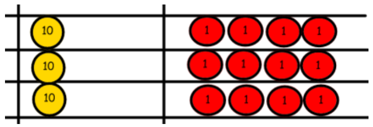
42 ÷ 3

42 ÷ 3=

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Finally move into decimal places to divide the total accurately.

$$\begin{array}{cc|ccccc} & & & 1 & 4 & . & 6 \\ & & & & {\scriptstyle 16} & & {\scriptstyle 21} \\ 3 & 5 & 5 & 1 & 1 & . & 0 \end{array}$$

Mental Method Strategies

Addition and Subtraction

Year 1

Children should understand when to and be able to apply these strategies:

- reorder numbers when adding, e.g. put the larger number first
- count on or back in ones, twos or tens
- partition small numbers, e.g. 8 + 3 = 8 + 2 + 1
- partition and combine tens and ones
- partition: double and adjust, e.g. 5 + 6 = 5 + 5 + 1

Year 2

Children should understand when to and be able to apply these strategies:

- reorder numbers when adding
- partition: bridge through 10 and multiples of 10 when adding and subtracting
- partition and combine multiples of tens and ones
- use knowledge of pairs making 10
- partition: count on in tens and ones to find the total
- partition: count on or back in tens and ones to find the difference
- partition: add a multiple of 10 and adjust by 1
- partition: double and adjust

Year 3:

Children should understand when to and be able to apply these strategies:

- reorder numbers when adding
- identify pairs totalling 10 or multiples of 10
- partition: add tens and ones separately, then recombine
- partition: count on in tens and ones to find the total
- partition: count on or back in tens and ones to find the difference

- partition: add or subtract 10 or 20 and adjust
- partition: double and adjust
- partition: count on or back in minutes and hours, bridging through 60 (analogue times)

Year 4

Children should understand when to and be able to apply these strategies:

- count on or back in hundreds, tens and ones
- partition: add tens and ones separately, then recombine
- partition: subtract tens and then ones, e.g. subtracting 27 by subtracting 20 then 7
- subtract by counting up from the smaller to the larger number
- partition: add or subtract a multiple of 10 and adjust, e.g. 56 + 29 = 56 + 30 – 1, or 86 – 38 = 86 – 40 + 2
- partition: double and adjust
- use knowledge of place value and related calculations, e.g. work out 140 + 150 = 290 using 14 + 15 = 29
- partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)

Year 5

Children should understand when to and be able to apply these strategies:

- count on or back in hundreds, tens, ones and tenths
- partition: add hundreds, tens or ones separately, then recombine
- subtract by counting up from the smaller to the larger number
- add or subtract a multiple of 10 or 100 and adjust
- partition: double and adjust
- use knowledge of place value and related calculations, e.g. 6.3 – 4.8 using 63 – 48
- partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)

Year 6

Children should understand when to and be able to apply these strategies:

- count on or back in hundreds, tens, ones, tenths and hundredths

- use knowledge of place value and related calculations, e.g. 680 + 430, 6.8 + 4.3, 0.68 + 0.43 can all be worked out using the related calculation 68 + 43
- use knowledge of place value and of doubles of two-digit whole numbers
- partition: double and adjust
- partition: add or subtract a whole number and adjust, e.g. 4.3 + 2.9 = 4.3 + 3 – 0.1, 6.5 – 3.8 = 6.5 – 4 + 0.2
- partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times, 12-hour and 24- hour clock)

<u>Mental Method Strategies</u>

<u>Multiplication and Division</u>

Year 1

Children should understand when to and be able to apply these strategies:

• use patterns of last digits, e.g. 0 and 5 when counting in fives

Year 2

Children should understand when to and be able to apply these strategies:

• partition: double the tens and ones separately, then recombine

• use knowledge that halving is the inverse of doubling and that doubling is equivalent to multiplying by two

• use knowledge of multiplication facts from the 2, 5 and 10 times-tables, e.g. recognise that there are 15 objects altogether because there are three groups of five

Year 3

Children should understand when to and be able to apply these strategies:

• partition: when doubling, double the tens and ones separately, then recombine

• partition: when halving, halve the tens and ones separately, then recombine

• use knowledge that halving and doubling are inverse operations

• recognise that finding a unit fraction is equivalent to dividing by the denominator and use knowledge of division facts

• recognise that when multiplying by 10 or 100 the digits move one or two places to the left and zero is used as a place holder

Year 4

Children should understand when to and be able to apply these strategies:

• partition: double or halve the tens and ones separately, then recombine

• use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right and zero is used as a place holder

• use knowledge of multiplication facts and place value,

e.g. 7 x 8 = 56 to find 70 x 8, 7 x 80

• use partitioning and the distributive law to multiply,

e.g. $13 \times 4 = (10 + 3) \times 4 = (10 \times 4) + (3 \times 4) = 40 + 12 = 52$

Year 5

Children should understand when to and be able to apply these strategies:

• multiply or divide by 4 or 8 by repeated doubling or halving

• form an equivalent calculation, e.g. to multiply by 5, multiply by 10, then halve; to multiply by 20, double, then multiply by 10

• use knowledge of doubles/ halves and understanding of place value, e.g. when multiplying by 50 multiply by 100 and divide by 2

• use knowledge of division facts, e.g. when carrying out a division to find a remainder

• use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right relative to the decimal point, and zero is used as a place holder

• use knowledge of multiplication and division facts and understanding of place value, e.g. when calculating with multiples of 10

• use knowledge of equivalence between fractions and percentages, e.g. to find 50%, 25% and 10%

• use knowledge of multiplication and division facts to find factor pairs

Year 6

Children should understand when to and be able to apply these strategies:

• partition: use partitioning and the distributive law to divide tens and ones separately,

e.g. 92 ÷ 4 = (80 + 12) ÷ 4 = 20 + 3 = 23

• form an equivalent calculation,

e.g. to divide by 25, divide by 100, then multiply by 4; to divide by 50, divide by 100, then double

• use knowledge of the equivalence between fractions and percentages and the relationship between fractions and division

• recognise how to scale up or down using multiplication and division,

e.g. if three oranges cost 24p: one orange costs 24 ÷ 3 = 8p four oranges cost 8 × 4 = 32p

• Use knowledge of multiplication and division facts to identify factor pairs and numbers with only two factors